The Squared Lorentz Factor in Quantum Computation: Grover Search, Quantum Batteries, and the Zeno Effect

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Abstract

The conformal equivalence $ds^2(BK) = 4\gamma^2\, ds^2(Bures)$ on the qubit Bloch ball, where $\gamma^2(V) = 1/(1 - V^2)$ is the squared Lorentz factor, provides a unified geometric account of fundamental limits in quantum computation. (1) Grover's search algorithm follows a Fubini-Study geodesic whose Fisher information cost equals arctanh(Vf), the Beltrami-Klein distance to the target. (2) Quantum battery charging power is bounded by the quantum Fisher information; for a two-level battery, maximum power scales as $\gamma^2(r)$, with a geometric distance ratio $gd(\eta)/(2\eta)$ mediated by the Gudermannian function that quantifies the conversion between Bures and BK distances. (3) The quantum Zeno effect freezes states under frequent measurement with a timescale proportional to $\gamma(V)$. All three limits arise from γ^2 as the exchange rate between statistical (Bures) and dynamical (Beltrami-Klein) distances on the Bloch ball. The conformal equivalence is derived self-consistently in Appendix A from standard metric expressions, with independent literature support from six research programs. The $\gamma^2 = 2$ threshold in the Zeno effect yields a conceptual prediction whose experimental realization requires continuous weak measurement protocols beyond the idealized projective model.

Keywords: Fisher information; Lorentz factor; Grover algorithm; quantum battery; Zeno effect; Bures metric; information geometry

1. Introduction

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Quantum computation operates on qubits whose state space is the Bloch ball B^3. Two natural Riemannian metrics inhabit this space: the Bures metric, which quantifies statistical distinguishability between quantum states [6], and the Beltrami-Klein metric, the unique constant-negative-curvature metric whose geodesics are chords of the ball. These metrics are conformally equivalent with conformal factor $\gamma^2(r) = 1/(1 - r^2)$ (see Appendix A for the self-contained derivation), and this conformal factor equals the Fisher information $I(V) = \gamma^2(V)$ of a binary measurement with visibility $V = r$.

Burns, Greenfield, and Dressel [7] independently arrived at the same conformal structure by showing that projective quantum measurement acts as a Lorentz boost on the Bloch ball, with the squared Lorentz factor governing the information gain. Halverson, Harvey, and Nee [14] proved that the Fisher information metric is the unique physically motivated metric on parameter space, providing a universality argument for why γ^2 appears across disparate quantum phenomena rather than being an artifact of a particular parametrization. The Bures metric is the minimal monotone metric in the Petz classification [15], selected by the conformal equivalence from the infinite family of quantum Fisher information metrics.

Deffner and Campbell [8] reviewed quantum speed limits arising from geometric considerations on state space. The present paper shows that three fundamental limits in quantum computation share a common geometric origin in the conformal factor γ^2: the query complexity of Grover search, the charging efficiency of quantum batteries, and the timescale of the quantum Zeno effect. Each limit encodes the exchange rate between finite Bures distances and diverging Beltrami-Klein (rapidity) distances on the Bloch ball.

2. Grover search as a Fisher geodesic

Grover's algorithm [2] finds a marked item in an unsorted database of N items using $O(\sqrt{N})$ queries. Cafaro, Felice, and Alsing [3] showed the algorithm evolves the state along a geodesic of the Fubini-Study metric on the two-dimensional subspace spanned by the uniform superposition $|s\rangle$ and the target state $|t\rangle$. Rossetti, Cafaro, and Alsing [9] further quantified geodesic efficiency on the Bloch sphere.

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The initial overlap $|(\langle s|t\rangle)| = 1/\sqrt{N}$ gives a final visibility $Vf = \sqrt{(1 - 1/N)}$. In the conformal framework, the Fubini-Study metric on the search subspace is the Bures metric on a qubit Bloch ball. The Fisher information cost of traversing the geodesic from V = 0 to V = Vf is

$$C(\text{Fisher}) = \int \gamma^2(V)\, dV = \text{arctanh}(Vf)$$

which is precisely the Beltrami-Klein distance to the target. The query count is determined by the Bures arc length (1/2)arcsin(Vf), and the ratio of BK distance to Bures arc length grows as $\gamma(Vf)$: each query pays a fixed Bures cost but an escalating Fisher cost as the target state is approached.

Geometric interpretation of Grover optimality. Bennett, Bernstein, Brassard, and Vazirani [10] proved Grover's algorithm is optimal via the polynomial method. In the conformal framework, this optimality admits a geometric interpretation: Grover's algorithm follows a geodesic of the Fubini-Study (Bures) metric, and geodesics minimize arc length among smooth curves connecting two points on a Riemannian manifold. Since the Fisher information cost C(Fisher) = arctanh(Vf) is a monotone function of the Bures arc length, the geodesic path also minimizes the Fisher cost among paths in the search subspace. This geometric interpretation does not constitute an independent proof of Grover optimality; rather, it shows that the known optimality result is naturally expressed in the language of the Bures-BK conformal equivalence. Departures from the geodesic (e.g., due to decoherence) would incur higher Fisher cost, consistent with the polynomial method bound.

3. Quantum battery charging

Julià-Farré et al. [4] proved the power bound $P \leq (1/2)\sqrt{(I_E \cdot \Delta E^2(B))}$, where IE is the Fisher information in the energy eigenspace. For a single-qubit battery, parametrize the state by Bloch radius r along the energy axis. The outcome probability of an energy measurement is $p = (1 + r)/2$, with visibility V = r. The Fisher information is $I_E = 1/(1 - V^2) = \gamma^2(V)$. This follows from the Bernoulli Fisher information $I(p) = 1/(p(1 - p))$ under the substitution $p = (1 + V)/2$ (see Appendix A for the full derivation). Gyhm, Šafránek, and Rosa [5] proved the quantum charging distance equals the Bures angle. Spehner [11] provided the Bures geodesic framework.

For a qubit battery charged from the maximally mixed state (r = 0) to Bloch radius rf, the energy cost in Beltrami-Klein units is arctanh(rf) while the information gained in Bures units is (1/2)arcsin(rf). We define the **geometric distance ratio**:

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$$R(\text{geometric}) = \arcsin(rf) / (2 \operatorname{arctanh}(rf)) = \operatorname{gd}(\eta) / (2\eta)$$

where $\eta = \operatorname{arctanh}(rf)$ is the rapidity. **Clarification:** This quantity is a purely geometric conversion factor between Bures distance and BK distance, not an operational thermodynamic efficiency. It quantifies what fraction of the BK-measured distance translates into Bures-measured distinguishability. It is not protocol-dependent and is not directly measurable as a thermodynamic efficiency in the laboratory sense. We retain the term 'geometric distance ratio' (rather than 'efficiency') throughout to avoid confusion with operational definitions.

This ratio decreases monotonically from 1/2 (as $rf \to 0$) to 0 (as $rf \to 1$). The Gudermannian function $\operatorname{gd}(\eta) = \arcsin(\tanh \eta)$ is bounded by $\pi/2$ while $\eta \to \infty$, so $\operatorname{gd}(\eta)/(2\eta) \to 0$: the geometric cost of approaching the pure-state boundary diverges.

4. The quantum Zeno effect and γ^2

The quantum Zeno effect, first formalized by Misra and Sudarshan [12], states that frequent measurement can freeze state evolution. Consider a qubit with energy splitting ΔE evolving under Hamiltonian H while being repeatedly measured in a basis tilted by angle θ from the energy eigenbasis. The survival probability after a short interval δt is

$$P(\delta t) = 1 - (\Delta E\ \delta t / \hbar)^2 \sin^2\theta + O(\delta t^4)$$

Writing $\sin^2\theta = 1 - V^2 = 1/\gamma^2(V)$, where $V = \cos \theta$ is the visibility of the measurement relative to the energy basis, the Zeno timescale is $\tau Z = (\hbar/\Delta E) \cdot \gamma(V)$.

4.1 Limiting cases

When $V = 0$ (measurement orthogonal to energy eigenbasis), $\gamma = 1$ and $\tau Z = \hbar/\Delta E$: minimum protection. When $V \to 1$ (aligned), $\gamma \to \infty$ and $\tau Z \to \infty$: trivially frozen because $[H, M] = 0$.

4.2 The $\gamma^2 = 2$ threshold

At $V = 1/\sqrt{2} \approx 0.707$, $\gamma^2 = 2$ and $\tau Z = \sqrt{2}\ \hbar/\Delta E$. Below this threshold, measurement-induced decoherence exceeds Zeno stabilization; above it, Zeno protection dominates. To achieve survival probability $P_n > 1 - \varepsilon$, one requires $n \gtrsim (\Delta E\ T / \hbar)^2 / (\varepsilon\ \gamma^2)$ measurements. At $\gamma^2 = 2$, this count is half the $V = 0$ baseline.

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4.3 Experimental considerations and model limitations

The $\gamma^2 = 2$ threshold is derived under the idealized assumptions of instantaneous projective measurement and unitary evolution between measurements. Real experiments on superconducting transmon qubits involve finite-time dispersive readout, where the qubit-resonator interaction implements a continuous weak measurement with measurement strength controlled by the dispersive shift χ and the readout drive amplitude. In this regime, several complications arise:

(i) **Finite measurement duration.** Each dispersive readout takes tens to hundreds of nanoseconds, during which the qubit continues to evolve. The projective limit ($\delta t \to 0$, $n \to \infty$) is never exactly realized. The Zeno effect in the weak measurement regime has been analyzed by Jacobs and Steck [16], who showed that continuous monitoring produces a modified Zeno dynamics where the measurement strength (not the measurement count) controls state freezing.

(ii) **Measurement back-action and dephasing.** Dispersive readout induces measurement-induced dephasing (Gambetta et al. [17]) that is not captured by the idealized projective model. The additional dephasing rate $\Gamma\varphi$ modifies the effective Zeno timescale and could obscure the predicted γ^2 dependence.

(iii) **State preparation and rotation errors.** Implementing a measurement at angle θ requires pre- and post-measurement rotations $Ry(\theta)$ with gate fidelity > 99.9% to avoid systematic bias in the survival probability.

The $\gamma^2 = 2$ crossover is therefore best understood as a **conceptual prediction of the conformal framework**: within the idealized projective Zeno model, the conformal factor directly determines the measurement overhead, with a sharp 2:1 ratio at $\theta = \pi/4$ vs. $\theta = \pi/2$. Experimental verification would require either (a) a continuous weak measurement protocol (e.g., circuit QED with variable dispersive coupling) analyzed within the Ito-Dechant stochastic Fisher information framework [13], where γ^2 enters as the leading-order contribution to the measurement back-action rate; or (b) a trapped-ion platform where projective measurements are faster relative to coherent evolution timescales. We do not claim that the idealized prediction is immediately testable on current hardware without accounting for these realistic effects.

5. Unified framework

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All three limits derive from $\gamma^2(V) = 1/(1 - V^2)$ as the exchange rate between statistical and dynamical descriptions of qubit evolution. Grover search: cost = arctanh(Vf) (BK distance), query count determined by Bures arc length, ratio grows as γ. Battery charging: maximum power $\propto \gamma^2$, geometric distance ratio = gd(η)/(2η), monotonically decreasing from 1/2 to 0. Zeno effect: timescale = $(\hbar/\Delta E) \cdot \gamma(V)$, protection strongest when measurement and dynamics are aligned, conceptual crossover at $\gamma^2 = 2$.

Quantum computation operates on the Bures manifold (finite statistical distances); thermodynamic and dynamical costs live on the Beltrami-Klein manifold (diverging rapidity distances). The conformal factor γ^2 is the exchange rate. The Gudermannian function gd(η) = arcsin(tanh η) is the map between these two descriptions.

6. Limitations

Five limitations require acknowledgment. First, the Grover analysis assumes an exact two-dimensional subspace; decoherence perturbs the geodesic and the Fisher cost acquires corrections of order $O(\gamma^2\varepsilon)$ where ε is the leakage rate. Second, the battery charging bound [4] is an upper bound on power, not an equality; the γ^2 scaling governs the bound but not necessarily achievable power in all protocols. Third, the Zeno analysis (Section 4) assumes projective measurement; for continuous weak measurement, the relationship to γ^2 requires the Ito-Dechant framework [13] and enters as the leading-order contribution, with corrections from finite measurement strength and dephasing. Fourth, the conformal equivalence $ds^2(\mathrm{BK}) = 4\gamma^2\, ds^2(\mathrm{Bures})$ is specific to qubits (N = 2); a Weyl tensor obstruction prevents extension to higher-dimensional systems [1]. Fifth, the geometric interpretation of Grover optimality (Section 2) is a reformulation of the Bennett et al. [10] result in conformal language, not an independent proof; it inherits the assumptions and scope of the polynomial method.

7. Conclusion

The squared Lorentz factor $\gamma^2(V) = 1/(1 - V^2)$ provides a unified geometric account of Grover search complexity, quantum battery charging, and the quantum Zeno effect. Each arises from the conformal equivalence between Bures and Beltrami-Klein metrics on the qubit Bloch ball, derived self-consistently in Appendix A. The Gudermannian function mediates between finite (Bures) and

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infinite (Beltrami-Klein) distances, providing the geometric origin of computational resource scaling. The $\gamma^2 = 2$ threshold in the Zeno effect yields a conceptual prediction whose experimental realization requires continuous weak measurement protocols on circuit QED or trapped-ion platforms.

Declarations

Generative AI: The author used Claude (Anthropic) for literature search and manuscript formatting. All physical reasoning, derivations, and novel claims are the author's own work.

Data availability: No data was used. All results are analytical.

Conflicts of interest: The author declares no competing interests.

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Appendix A: Definitions, Conventions, and Self-Contained Derivation

A.1 The Bures metric on the qubit Bloch ball

The state of a qubit is represented by a density matrix $\rho = (1/2)(I + \vec{r} \cdot \vec{\sigma})$, where $\vec{r}$ is the Bloch vector with $|\vec{r}| = r \in [0, 1]$ and $\vec{\sigma}$ are the Pauli matrices. In spherical coordinates (r, θ, φ), the Bures metric (equivalently, the SLD quantum Fisher information metric) on the Bloch ball is [6, 15]:

$$ds^2(\text{Bures}) = dr^2 / [4(1 - r^2)] + (r^2/4)(d\theta^2 + \sin^2\theta \, d\varphi^2)$$

This metric has constant positive sectional curvature $K = +4$ (a 3-sphere of radius 1/2). The radial distance from the center to radius r is $d(\text{Bures})(0, r) = (1/2)\arcsin(r)$.

A.2 The Beltrami-Klein metric

The Beltrami-Klein model of hyperbolic space on the unit ball is:

$$ds^2(\text{BK}) = dr^2 / (1 - r^2)^2 + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) / (1 - r^2)$$

This metric has constant negative sectional curvature $K = -1$. The radial distance is $d(\text{BK})(0, r) = \operatorname{arctanh}(r)$, which diverges as $r \to 1$.

A.3 The conformal equivalence

Direct computation. Comparing term by term:

Radial: $ds^2(\text{BK, radial}) = dr^2/(1 - r^2)^2 = [4/(1 - r^2)] \times [dr^2/(4(1 - r^2))] = 4\gamma^2(r) \times ds^2(\text{Bures, radial})$

Angular: $ds^2(\text{BK, angular}) = r^2 \, d\Omega^2/(1 - r^2) = [4/(1 - r^2)] \times [r^2 \, d\Omega^2/4] = 4\gamma^2(r) \times ds^2(\text{Bures, angular})$

Therefore: $\mathbf{ds^2(BK) = 4\gamma^2(r) \, ds^2(Bures)}$ where $\gamma^2(r) = 1/(1 - r^2)$.

This is a conformal equivalence between a spherical geometry (Bures, $K = +4$) and a hyperbolic geometry (BK, $K = -1$). In dimension 3, both metrics have vanishing Weyl tensor (by dimensional necessity), and constant sectional curvature implies vanishing Cotton tensor, so the conformal equivalence is geometrically natural. For $N \geq 3$ quantum systems (state space dimension $d = N^2 - 1 \geq 8$), the Bures metric has nonconstant sectional curvature and nonvanishing Weyl tensor (300 independent components for qutrits), obstructing any such conformal equivalence [1].

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A.4 Fisher information identification

For a binary measurement with outcome probability $p = (1 + V)/2$, the Fisher information with respect to V is:

$$I(V) = (dp/dV)^2 / [p(1 - p)] = (1/2)^2 / [(1 + V)(1 - V)/4] = 1/(1 - V^2) = \gamma^2(V)$$

Setting $r = V$ identifies the Bloch radius with the measurement visibility, and the conformal factor with the Fisher information: $\gamma^2(r) = I(V)$.

A.5 Independent literature support

The conformal structure γ^2 on the Bloch ball is not derived solely from [1]. Six independent research programs converge on it: (i) Burns, Greenfield, and Dressel [7] proved measurement backaction is a Lorentz boost via $SL(2,\mathbb{C})$; (ii) Braunstein and Caves [6] established the SLD Fisher metric on qubit states; (iii) the Stokes-Minkowski formalism identifies polarization degree with velocity; (iv) Chen and Ungar identified the Bloch ball with Einstein velocity addition; (v) the Chentsov-Petz theorem [15] establishes uniqueness of the Fisher-Rao metric; (vi) Halverson, Harvey, and Nee [14] proved the Fisher metric is the unique natural metric on parameter space.

A.6 The Gudermannian bridge

The radial Bures distance $d(\text{Bures})(0, r) = (1/2)\arcsin(r)$ and BK distance $d(\text{BK})(0, r) = \operatorname{arctanh}(r)$ are related by $d(\text{Bures}) = (1/2)\operatorname{gd}(d(\text{BK}))$, where gd is the Gudermannian function $\operatorname{gd}(\eta) = \arcsin(\tanh \eta)$. This is verified by: $(1/2)\arcsin(r) = (1/2)\arcsin(\tanh(\operatorname{arctanh}(r))) = (1/2)\operatorname{gd}(\operatorname{arctanh}(r))$, since $\tanh(\operatorname{arctanh}(r)) = r$.

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