Discriminating Thomas–Wigner from Pancharatnam–Berry Rotations in Sequential Weak Qubit Measurements

A Variable-Visibility Protocol

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A methods paper presenting a pre-registered experimental protocol that discriminates between the Thomas–Wigner and Pancharatnam–Berry predictions for sequential non-collinear weak measurements on a qubit, with quantitative falsification thresholds achievable on existing superconducting, trapped-ion, and neutral-atom hardware.

Abstract

The Fisher–Lorentz identity $I(V) = \gamma^2(V) = 1/(1 - V^2)$ for binary quantum measurements predicts that sequential non-collinear weak measurements on a qubit accumulate a Thomas–Wigner (TW) rotation with a specific, visibility-dependent closed form. This prediction is numerically distinct from the Pancharatnam–Berry (PB) geometric phase that governs the same measurement cycle under the standard pure-state geometric-phase interpretation. We present a pre-registered experimental protocol that discriminates between the two predictions on existing superconducting, trapped-ion, and neutral-atom platforms by sweeping the measurement visibility $V \in [0.3, 0.99]$. For the octant spherical triangle $\hat{z} \to \hat{x} \to \hat{y} \to \hat{z}$, the PB prediction is $\Omega_{\mathrm{PB}} = 45^\circ$ (fixed by the enclosed solid angle of $\pi/2$ sr), while Ω_{TW} varies from 2.70° at $V = 0.30$ to 73.94° at $V = 0.99$, crossing the PB baseline at $V_\times \approx 0.9102$. The signed discrepancy $\Omega_{\mathrm{TW}} - \Omega_{\mathrm{PB}}$ ranges from -42° to $+29^\circ$ across the sweep and provides two independent discrimination landmarks: a monotonic V-dependence and a crossing point. We specify shot counts, tomographic precision, readout fidelity, and systematic-error budgets sufficient for 3σ discrimination. The protocol requires no new hardware; a successful run on any reasonable-fidelity qubit platform either confirms a central prediction of the Fisher–Lorentz framework or falsifies it at 3σ. This paper is a methods paper: its purpose is to enable the experiment, not to re-establish the theory.

Keywords: Thomas–Wigner rotation, Pancharatnam–Berry phase, weak measurement, qubit tomography, Fisher information, squared Lorentz factor, geometric phase, $\mathrm{SL}(2, \mathbb{C})$ measurement structure, pre-registered protocol, superconducting qubit, trapped ion, neutral atom

1 Introduction

Sequential non-collinear measurements on a quantum system accumulate a geometric phase. For pure states traversing a closed loop in Hilbert space, this phase is the Pancharatnam–Berry phase [1, 2]. For mixed states undergoing partial (unsharp) measurements, the relevant object is the Thomas–Wigner rotation arising from the composition of non-collinear Lorentz boosts under the qubit measurement group $\mathrm{SL}(2,\mathbb{C}) \cong \mathrm{Spin}^+(1,3)$, the double cover of the restricted Lorentz group $\mathrm{SO}^+(1,3)$ [3, 4, 10]. The central conjecture of the Fisher–Lorentz framework [5, 6] is that these two descriptions differ at finite measurement visibility in a specific, quantitative way that is experimentally accessible on current hardware.

This paper does not argue for the Fisher–Lorentz framework. It assumes the framework's prediction, works out its numerical consequences, specifies an experiment that can confirm or refute the prediction at 3σ on existing hardware, and pre-registers the expected outcomes and falsification thresholds. The audience is experimentalists working on superconducting qubits, trapped ions, neutral atoms, or cold-neutron interferometry. The purpose is to enable a decisive measurement.

Relation to existing measurement-induced geometric-phase literature. Gebhart et al. [11] and Cho et al. [12] established that measurement-induced geometric phases under sequential weak measurements carry visibility dependence in pure-state settings, and demonstrated topological transitions at specific measurement strengths. The present work contributes the specific closed form of Eq. (2) derived from $\mathrm{SL}(2,\mathbb{C})$ boost composition on the mixed-state Bloch ball [3], and extracts from it the crossing-point prediction at $V_\times \approx 0.9102$ as an operational discriminator usable as a pre-registered falsification test. The protocol is hardware-agnostic and requires only tunable measurement strength plus single-qubit tomography.

Organization. Section 2 states the two theoretical predictions and the regime in which they differ. Section 3 specifies the experimental protocol. Section 4 provides platform-specific shot-count and fidelity requirements. Section 5 pre-registers the predictions and falsification thresholds. Section 6 addresses interpretation of possible outcomes.

2 Theoretical Predictions

We consider two sequential binary measurements on a qubit initially prepared in a state $\rho_0 = (\mathbb{I} + r_0\,\hat{\mathbf{n}}_0 \cdot \boldsymbol{\sigma})/2$ with Bloch magnitude $r_0 \in [0,1]$. Each measurement has visibility $V \in (0,1)$ and is performed along a unit axis $\hat{\mathbf{n}}_k$ for $k = 1,2$, with angular separation $\theta = \arccos(\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2)$. The primary protocol uses the orthogonal configuration $\hat{\mathbf{n}}_0 = \hat{z}$, $\hat{\mathbf{n}}_1 = \hat{x}$, $\hat{\mathbf{n}}_2 = \hat{y}$, so that $\theta = \pi/2$.

2.1 Thomas–Wigner Prediction

Each measurement with visibility V along axis $\hat{\mathbf{n}}$ admits an $\mathrm{SL}(2,\mathbb{C})$ lift

$$L(\eta, \hat{\mathbf{n}}) = \cosh(\eta/2)\,\mathbb{I} + \sinh(\eta/2)\,(\hat{\mathbf{n}} \cdot \boldsymbol{\sigma}), \qquad \eta = \operatorname{arctanh}(V), \tag{1}$$

which is a Lorentz boost with rapidity η along $\hat{\mathbf{n}}$ [3]. The composition L_2L_1 polar-decomposes as a positive boost times a unitary rotation $U = \cos(\Omega/2)\,\mathbb{I} - i\sin(\Omega/2)\,(\hat{\mathbf{u}}\cdot\boldsymbol{\sigma})$ with axis $\hat{\mathbf{u}} = (\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2)/|\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2|$ and angle

$$\boxed{\tan\left(\frac{\Omega_{\mathrm{TW}}}{2}\right) = \frac{V_1V_2\,\gamma_1\gamma_2\sin\theta}{(\gamma_1+1)(\gamma_2+1) + V_1V_2\,\gamma_1\gamma_2\cos\theta}} \tag{2}$$

where $\gamma_k = 1/\sqrt{1-V_k^2}$. Equation (2) is the standard Thomas–Wigner rotation angle for non-collinear boost composition [13, 14, 15], expressed in the visibility variables natural to qubit measurements [7, 8]. A convenient hyperbolic-half-angle identity

$$\tanh(\eta/2) = \frac{V}{1+\gamma^{-1}} = \frac{V\gamma}{\gamma+1} \tag{3}$$

makes Eq. (2) equivalent to the symmetric form $\tan(\Omega/2) = \tanh(\eta_1/2)\tanh(\eta_2/2)\sin\theta\big/[1+\tanh(\eta_1/2)\tanh(\eta_2/2)\cos\theta]$. For equal visibility $V_1 = V_2 = V$ at $\theta = \pi/2$:

$$\tan\left(\frac{\Omega_{\mathrm{TW}}}{2}\right) = \tanh^2(\eta/2), \qquad \eta = \operatorname{arctanh}(V). \tag{4}$$

2.2 Pancharatnam–Berry Prediction

The Pancharatnam–Berry phase for a pure state traversing a closed loop on the Bloch sphere enclosing solid angle Ω_{solid} is $\Phi_{\mathrm{PB}} = \Omega_{\mathrm{solid}}/2$ [1]. For sequential projective measurements along $\hat{\mathbf{n}}_0, \hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2$ (plus closing leg to $\hat{\mathbf{n}}_0$) bounding a spherical triangle with internal angles α, β, γ, Girard's theorem gives $\Omega_{\mathrm{solid}} = \alpha+\beta+\gamma-\pi$, so $\Phi_{\mathrm{PB}} = (\alpha+\beta+\gamma-\pi)/2$. Under sequential projective measurements, the Bloch-sphere rotation that tracks this phase is $\Omega_{\mathrm{PB}} = \Phi_{\mathrm{PB}}$ [11, 12].

For the specific geometry used in this protocol — $\hat{\mathbf{n}}_0 = \hat{z}$, $\hat{\mathbf{n}}_1 = \hat{x}$, $\hat{\mathbf{n}}_2 = \hat{y}$, with closing leg back to $\hat{z}$ — the measurement trajectory traces the *octant spherical triangle* with three right-angle vertices. Its solid angle is $\Omega_{\mathrm{solid}} = 3\pi/2 - \pi = \pi/2$ steradians, giving

$$\boxed{\Omega_{\mathrm{PB}} = \frac{\Omega_{\mathrm{solid}}}{2} = \frac{\pi}{4} = 45^\circ} \tag{5}$$

independent of visibility V. This visibility-independence is the key discriminator: any V-dependence observed in the measured rotation angle falsifies the pure-state PB prediction and corroborates the mixed-state TW prediction.

Remark. Throughout this paper, Ω_{PB} denotes the rotation angle on the Bloch sphere predicted by the standard Pancharatnam–Berry geometric phase under sequential projective measurements, which for pure states equals half the solid angle enclosed by the measurement trajectory [11, 12]. The protocol tomographically measures this rotation directly, sidestepping any interpretive distinction between the acquired phase and the rotation angle.

2.3 Where the Two Predictions Differ

1. **Weak-measurement limit $V \to 0$:** $\Omega_{\mathrm{TW}} \to 0$ (no boost, no rotation), while $\Omega_{\mathrm{PB}} = 45^\circ$ remains fixed by solid angle. Maximum negative discrepancy $\Delta \to -45^\circ$.

2. **Crossing point $V_\times \approx 0.9102$:** Ω_{TW} transits through 45° exactly. Here the two predictions coincide in magnitude; in this narrow band, discrimination relies on the derivative $\mathrm{d}\Omega/\mathrm{d}V$ rather than the instantaneous value.
3. **Projective limit $V \to 1$:** $\Omega_{\mathrm{TW}} \to \theta = 90^\circ$, while Ω_{PB} remains at 45°. The two predictions *do not* coincide at $V = 1$; the TW curve exceeds the PB baseline by 45°.
4. **Intermediate V:** Ω_{TW} increases monotonically from 0° toward 90° while $\Omega_{\mathrm{PB}} = 45^\circ$ is fixed. The observable signature is a monotonic V-dependence of the measured rotation angle crossing the PB baseline at $V_\times$, providing two independent discrimination landmarks.

Numerical table. Table 1 lists Ω_{TW} and the signed discrepancy $\Delta = \Omega_{\mathrm{TW}} - \Omega_{\mathrm{PB}}$ for equal-visibility measurements at $\theta = \pi/2$, with the octant-geometry PB baseline $\Omega_{\mathrm{PB}} = 45^\circ$.

Table 1: Thomas–Wigner rotation angle and signed discrepancy $\Delta = \Omega_{\mathrm{TW}} - \Omega_{\mathrm{PB}}$ relative to the octant-geometry Pancharatnam–Berry baseline $\Omega_{\mathrm{PB}} = 45^\circ$. All TW values computed from Eq. (2) at equal visibility $V_1 = V_2 = V$ and $\theta = \pi/2$.

V	γ	γ^2	Ω_{TW} [deg]	Δ [deg]
0.30	1.0483	1.0989	2.7008	−42.2992
0.50	1.1547	1.3333	8.2132	−36.7868
0.60	1.2500	1.5625	12.6804	−32.3196
0.70	1.4003	1.9608	18.9355	−26.0645
0.80	1.6667	2.7778	28.0725	−16.9275
0.90	2.2942	5.2632	42.8962	−2.1038
0.95	3.2026	10.2564	55.3181	10.3181
0.99	7.0888	50.2513	73.9408	28.9408

The signed discrepancy $|\Delta|$ exceeds 10° across most of the sweep range and is resolvable with modest shot counts on any current qubit platform. The sign flip of Δ at $V_\times \approx 0.9102$ is an independent qualitative discriminator: a single measurement run that captures points on both sides of $V_\times$ falsifies either prediction by the sign of the excursion alone.

3 Experimental Protocol

3.1 State Preparation

Prepare the qubit in a known mixed state $\rho_0 = (\mathbb{I} + r_0\,\hat{\mathbf{n}}_0 \cdot \boldsymbol{\sigma})/2$ with Bloch magnitude $r_0 \in [0.3, 0.9]$ and axis $\hat{\mathbf{n}}_0 = \hat{z}$. Mixed-state preparation can be achieved by partial decoherence, ensemble averaging over random phases, or explicit Kraus-operator preparation. The exact value of r_0 is not critical; it sets the upper bound on the achievable effective visibility but does not bias the V-dependence of Ω.

3.2 Sequential Weak Measurement

Perform two sequential partial measurements:

- **Measurement 1:** along $\hat{\mathbf{n}}_1 = \hat{x}$ with strength V (controlled via measurement duration τ_1, dispersive coupling χ, or laser detuning; platform-specific).
- **Measurement 2:** along $\hat{\mathbf{n}}_2 = \hat{y}$ (orthogonal to $\hat{\mathbf{n}}_1$) with strength V.

Both measurements are implemented by symmetric Kraus operators $K_\pm = \sqrt{(\mathbb{I} \pm V\,\hat{\mathbf{n}} \cdot \boldsymbol{\sigma})/2}$. Record both measurement outcomes. Keep the qubit state conditional on both outcomes (no averaging over the outcome pairs yet).

3.3 Tomographic Readout

After the second measurement, perform full single-qubit state tomography on the conditional post-measurement state $\rho_{\pm\pm}$ for each outcome pair $(\pm, \pm)$. Standard procedure [9]: measure $\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle$ via three separate projective measurements on independently-prepared ensembles conditioned on each outcome pair.

3.4 Rotation Angle Extraction

The conditional post-measurement Bloch vector $\mathbf{r}_{\pm\pm}$ differs from the naive boost-projected vector by the TW rotation. Writing

$$\mathbf{r}_{\pm\pm} = R(\Omega, \hat{\mathbf{u}}) \cdot \mathbf{r}^{\pm\pm}_{\text{boost}}, \tag{6}$$

where $\mathbf{r}^{\pm\pm}_{\text{boost}}$ is the Bloch vector that would result from two sequential boosts without rotation, and $R(\Omega, \hat{\mathbf{u}})$ is a rotation by angle Ω around axis $\hat{\mathbf{u}} = \hat{z}$ (determined by the chosen geometry $\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2 = \hat{z}$), extract Ω by fitting the measured $\mathbf{r}_{\pm\pm}$ to the rotation model using standard least-squares techniques.

Crucial control. Repeat the protocol at the highest achievable measurement strength $V \geq 0.999$ as a projective-limit calibration baseline. In this limit, $\Omega_{\text{TW}} \to \theta = \pi/2$, so the measured rotation should approach 90°; any residual deviation quantifies systematic errors (axis misalignment, readout error, finite measurement-strength leakage) and must be subtracted before comparing to the V-sweep data. Note that $V = 0.99$ gives $\Omega_{\text{TW}} = 73.94°$, still 16° below the limit, so calibration requires $V \geq 0.999$ (yielding $\Omega_{\text{TW}} \approx 84.88°$) or better.

4 Platform Requirements and Shot-Count Calculations

4.1 Superconducting Transmons

Typical parameters. Readout fidelity $\mathcal{F}_{\text{ro}} \geq 0.98$; $T_1 \geq 100\,\mu\text{s}$; $T_2^* \geq 30\,\mu\text{s}$; dispersive coupling $\chi/2\pi \geq 1\,\text{MHz}$. Major superconducting platforms (IBM, Google, Rigetti, OQC, IQM) meet these thresholds.

Measurement strength control. Visibility V is controlled by readout pulse duration τ: $V(\tau) = \tanh(\chi\,\eta_{\text{eff}}\,\tau)$ where η_{eff} is the measurement efficiency. To sweep $V \in [0.3, 0.99]$, vary τ over approximately a decade.

Shot-count requirement. Tomographic precision $\sigma_{\mathbf{r}} \sim 1/\sqrt{N}$ per Bloch-vector component, where N is shots per tomography setting. The mapping from Bloch-vector precision to rotation-angle precision depends on the Bloch magnitude: $\sigma_\Omega \approx \sigma_{\mathbf{r}}/|\mathbf{r}|$ in the small-angle linearization, so lower initial purity requires proportionally more shots. Taking $|\mathbf{r}| \approx 1$ (near-pure initial state) for the baseline estimate: for 3σ discrimination at the minimum-discrepancy regime near the crossing $V \approx 0.9$ (where $|\Delta| \approx 2°$–$10°$), we require $\sigma_\Omega \leq 3°$, mapping to $\sigma_{\mathbf{r}} \leq 0.05$ and $N \geq 400$ shots per tomography setting. With 3 Bloch components × 4 outcome pairs = 12 settings per V value, this gives $N_V \approx 4800$ shots per V setting. Sweeping 8 V values plus the calibration run: $N_{\text{total}} \approx 45\,\text{k}$ shots. Achievable in under one minute on any current superconducting platform. For mixed-state preparation with $|\mathbf{r}| \approx 0.5$, multiply by $\approx 4\times$ in the crossing region (giving $\sim 180\,\text{k}$ total shots, still a few-minute experiment). Away from the crossing (where $|\Delta| \geq 15°$), a coarser sweep with $N \geq 100$ per setting ($\sim 12\,\text{k}$ total shots) suffices to establish the monotonic V-dependence at 3σ.

4.2 Trapped Ions

Typical parameters. Readout fidelity $\mathcal{F}_{\text{ro}} \geq 0.995$; coherence times $T_2 \geq 1\,\text{s}$. Major platforms (Quantinuum H1/H2, IonQ) meet these thresholds. Partial measurements are implemented via tuned laser pulse amplitude controlling the state-dependent fluorescence rate.

Shot-count requirement. With higher readout fidelity than transmons, $\sim 25\,\text{k}$ shots total across the 8-V sweep plus calibration suffice for 3σ discrimination. Trapped-ion cycle times are slower (10 ms–100 ms per shot), giving 10–30 minute experiment times per full sweep.

4.3 Neutral Atom Rydberg Arrays

Typical parameters. Site-resolved fluorescence with $\mathcal{F}_{\text{ro}} \geq 0.99$ per atom; parallel operation across ~ 100 atoms. Platforms include QuEra, Atom Computing, and Pasqal.

Advantage. Parallelism: measure Ω simultaneously on many atoms at the same V setting. Effective shot count multiplied by array size. Total experimental time potentially shorter than trapped-ion by an order of magnitude.

4.4 Cold Neutron Interferometry (Palge et al. Proposal)

Palge et al. [8] present an independent proposal for observing Thomas–Wigner rotation in cold-neutron spin interferometry at non-relativistic speeds. This is a distinct platform from qubit hardware and provides a complementary falsification route for the same underlying geometric phenomenon. The present protocol focuses on qubit platforms for simplicity; the cold-neutron adaptation is documented in [8].

5 Pre-Registered Predictions and Falsification Thresholds

We pre-register the following quantitative predictions. Each is testable at 3σ with the shot counts specified in Section 4.

Prediction 1 (TW V-dependence)**.** *The measured rotation angle* $\Omega(V)$ *varies monotonically with* V*, increasing from* $\Omega \to 0$ *as* $V \to 0$ *to* $\Omega \to \theta = 90^\circ$ *as* $V \to 1$*, following Eq.* (2) *within* 3σ *tolerance. In particular, the sign of* $\Omega(V) - 45^\circ$ *flips from negative to positive at* $V_\times \approx 0.9102$*.*

Prediction 2 (Closed form)**.** *At* $\theta = \pi/2$ *and equal visibility* $V_1 = V_2 = V$*, the measured* Ω *values fit Eq.* (4) *across* $V \in [0.3, 0.99]$*. With the shot budget of Section 4 giving* $\sigma_\Omega \approx 3^\circ$*:* $\Omega(V{=}0.30) = 2.7(30)^\circ$*;* $\Omega(V{=}0.50) = 8.2(30)^\circ$*;* $\Omega(V{=}0.70) = 18.9(30)^\circ$*;* $\Omega(V{=}0.90) = 42.9(30)^\circ$*;* $\Omega(V{=}0.95) = 55.3(30)^\circ$*;* $\Omega(V{=}0.99) = 73.9(30)^\circ$*. Tighter predictions (*$\pm 1^\circ$*) are achievable with a* $10\times$ *shot-count increase.*

Prediction 3 (Crossing point)**.** *The measured rotation angle crosses the PB baseline* $\Omega_{\mathrm{PB}} = 45^\circ$ *at* $V_\times = 0.9102 \pm 0.01$ *under ideal calibration. To accommodate measurement-strength systematics of up to* $\pm 3\%$ *typical of current platforms, observation of a crossing anywhere in the wider acceptance window* $V \in [0.88, 0.94]$ *corroborates the TW prediction; absence of any crossing across the full sweep falsifies it.*

Prediction 4 (Calibration consistency)**.** *In the near-projective limit* $V \geq 0.999$*,* $\Omega \to \theta = 90^\circ$ *within systematic-error bounds; deviations quantify experimental systematics and must be less than* 5° *for the discrimination to be trustworthy. Note that* $V = 0.99$ *yields only* $\Omega_{\mathrm{TW}} = 73.94^\circ$ *(theoretical), so* $V \geq 0.999$ *is required for this calibration to reach the* 90° *baseline within tolerance.*

Falsification thresholds.

- If measured $\Omega(V)$ is V-independent and consistent with 45° across the sweep, PB holds; TW is falsified.
- If measured $\Omega(V)$ agrees with Eq. (2) at 3σ across the sweep (including the sign flip at $V_\times$), TW holds; PB is falsified for unsharp measurements.
- If neither: a new regime not covered by either prediction is revealed, motivating a more refined theoretical analysis.

6 Expected Outcomes and Interpretation

The protocol has three possible outcomes.

Outcome 1: TW prediction confirmed. The measured $\Omega(V)$ follows Eq. (2), including the sign flip at $V_\times$. This confirms the central experimental prediction of the Fisher–Lorentz framework [5]. It establishes that the qubit measurement process is operationally equivalent to a Lorentz boost at finite strength, and that the conformal equivalence $ds^2_{\mathrm{BK}} = 4\gamma^2\, ds^2_{\mathrm{Bures}}$ has measurable consequences in the rotation algebra of sequential measurements.

Outcome 2: PB prediction confirmed, TW falsified. The measured $\Omega(V)$ is V-independent at 45°. This falsifies a key prediction of the Fisher–Lorentz framework and constrains the interpretation of the $I(V) = \gamma^2$ identity to its algebraic and informational content, excluding the rotation-algebra reading. The framework's remaining claims (conformal equivalence, Weyl obstruction, Cramér–Rao saturation) survive.

Outcome 3: Neither prediction fits. The measured $\Omega(V)$ shows V-dependence but does not follow Eq. (2). This indicates new physics not captured by either the PB or TW frameworks in their current forms and motivates a more refined theoretical analysis.

In all three outcomes, the experiment produces a decisive scientific result. The protocol is therefore a win-win from the experimental perspective: no outcome is uninformative.

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Declarations

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Competing Interests
The author has filed US provisional patent applications related to the Fisher–Lorentz framework. No competing interests otherwise.

Data Availability
All predictions are derived analytically. A Python reproduction script computing Table 1 and Eq. (2) at arbitrary (V_1, V_2, θ) is available from the author on request.

AI Tools The author used Claude (Anthropic) as an AI assistant for editorial tasks, including language polishing, literature search support, and checking of mathematical expressions. All theoretical ideas, physical reasoning, derivations, and intellectual content are entirely the author's own work. The author has verified all results independently and takes full responsibility for the accuracy and integrity of all content in this manuscript.

Author Contributions
Single-author paper.

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