

The Algebra of the Qubit

Fourteen Theorems on $I(V) = \gamma^2(V)$

Bharath G. Srivats
Independent Researcher
bharathsrivats@outlook.com

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Based on: B. G. Srivats, "Fisher Information Is the Squared Lorentz Factor:
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¹Methods derived from this framework for qubit calibration drift detection, adaptive measurement-axis alignment, and Planckian-visibility sensor apparatus are the subject of pending US provisional patent applications.

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Fourteen Theorems on $I(V) = \gamma^2(V)$

Bharath G. Srivats
 Independent Researcher
 bharathsrivats@outlook.com
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Based on the mathematical identity proven in: B. G. Srivats, “Fisher Information Is the Squared Lorentz Factor: Why the Qubit Is Special,” Zenodo (2026). DOI: 10.5281/zenodo.19363449.

Abstract. This document collects fourteen theorems extending the Fisher information identity $I(V) = \gamma^2(V) = 1/(1 - V^2)$. Theorems T1 through T4 concern the internal mathematical structure of the identity: uniqueness of the Jeffreys prior as a reparameterization-covariant half-density (T1), closed-form Thomas–Wigner rotation for sequential binary measurements (T2), symmetric logarithmic derivative saturation (T3), and uniqueness of the Gudermannian bridge (T4). T5 establishes the closed-form retention function $R(r)$ for binary coarse-graining on mixed qubit states. T6 establishes via Lawvere’s fixed-point theorem that no observer embedded in a conformal information-geometric manifold can surjectively reconstruct the γ^2 field from within. T7 derives the energy–precision bound $E \cdot \text{Var}(\hat{\theta}) \geq kT \ln 2/\gamma^2(V)$, saturating the Landauer bound at $V = 0$. T8 establishes form-invariance of the conformal equivalence $ds_{\text{BK}}^2 = 4\gamma^2 ds_{\text{Bures}}^2$ under the Möbius action. T9 classifies unsharp binary qubit effects up to $\text{SU}(2)$ equivalence by visibility V . T10 identifies modular flow on qubits with $\text{SL}(2, \mathbb{C})$ boost at spinor rapidity $\zeta = 2it\eta$. T11 gives the exact SLOCC transformation law for single-qubit binary visibility. T12 establishes a scaling correspondence between holographic subregion complexity and Fisher information under boost, with dimension-specific constants κ_d ; the stronger identity $\kappa_d = 1$ is stated as a conjecture. T13 derives a γ^2 Page curve for black hole evaporation as a proof sketch. T14 proposes 3+1D spacetime selection from binary measurement structure via exceptional isomorphisms, acknowledged as a partial result with an explicit gap. Complete proofs are provided for T1 through T11. T12 and T13 are proof sketches. T14 is a partial result with an explicit gap acknowledged.

Keywords: Fisher information, Lorentz factor, Bures metric, Beltrami–Klein metric, qubit, conformal equivalence, Thomas–Wigner rotation, Lawvere fixed-point theorem, Landauer bound, holographic complexity, Page curve.

Conventions and Notation

- $V \in (-1, 1)$: visibility coordinate of a binary quantum measurement, $V = 2p - 1$.
- $\gamma(V) = 1/\sqrt{1 - V^2}$: Lorentz factor.
- $\gamma^2(V) = 1/(1 - V^2)$: squared Lorentz factor = Fisher information $I(V)$.
- $\eta = \text{arctanh}(V)$: rapidity; $V = \tanh(\eta)$; $\gamma = \cosh(\eta)$.
- Qubit Bloch-ball Bures metric: $ds_{\text{Bures}}^2 = dr^2/[4(1 - r^2)] + (r^2/4) d\Omega^2$.
- Beltrami–Klein metric: $ds_{\text{BK}}^2 = dr^2/(1 - r^2)^2 + r^2 d\Omega^2/(1 - r^2)$.

- Conformal equivalence: $ds_{\text{BK}}^2 = 4\gamma^2(r) \cdot ds_{\text{Bures}}^2$ (holds for $N = 2$; Weyl obstruction for $N \geq 3$).
- $\text{SL}(2, \mathbb{C}) \cong \text{Spin}^+(1, 3)$: double cover of the identity component of the 3 + 1D Lorentz group.

Related Work and Priority Statement

Burns, Greenfield, and Dressel [2] established that diffusive monitoring of a qubit generates $\text{SL}(2, \mathbb{C}) \cong \text{SO}^+(1, 3)$ dynamics. Fullwood, Vedral, and Guzmán-González [3] independently derived $\text{SO}^+(1, 3)$ on single qubits via linear-entropy preservation; Fullwood and Vedral [4] showed that sequential Pauli measurements reproduce Euclidean 3-space geometry. Szangolies [18] showed that multi-qubit entanglement yields $\text{SO}^+(5, 1)$ and $\text{SO}^+(9, 1)$ for two and three qubits respectively. Daryaei Goki, Ali-Akbari, Lezgi, and Esrafilian [5] showed that holographic subregion complexity of a boosted black brane diverges universally as γ^2 in $d = 2, 3, 4$ AdS/CFT. Gómez [6] connected quantum Fisher information to the Page curve via QFI scaling as $1/S_{\text{BH}}$.

The present work is the Fisher-information completion and unification of these results. The identity $I(V) = \gamma^2(V)$ was first established in the author's anchor preprint [1]. The conformal equivalence and Weyl obstruction are original to [1]. The present theorem pack extends that framework with fourteen theorems.

Part A: Internal Structure Theorems

T1. Uniqueness of the Jeffreys Prior

Theorem 1 (T1). *On the 1-dimensional binary Fisher–Rao manifold (M, g) with coordinate $V \in (-1, 1)$ and metric $g(V) dV^2 = \gamma^2(V) dV^2$, the Jeffreys prior $\pi_J(V) = \gamma(V)/\pi$ is the unique normalized smooth probability density proportional to the Riemannian volume form $\omega_g = \gamma(V) dV$. Equivalently, its associated half-density $\pi_J^{1/2}(dV)^{1/2}$ is the canonical half-density of g , covariant under every smooth reparameterization of M .*

Proof. The Riemannian volume density on (M, g) is $\omega_g = \sqrt{g(V)} dV = \gamma(V) dV$. Any density proportional to ω_g has the form $f = c\gamma$ for $c > 0$; normalization $\int_{-1}^1 f dV = c \int_{-1}^1 \gamma(V) dV = c\pi = 1$ fixes $c = 1/\pi$, giving $\pi_J(V) = \gamma(V)/\pi$. The half-density $\pi_J^{1/2}(dV)^{1/2} = \sqrt{\gamma(V)/\pi} (dV)^{1/2}$ coincides with the canonical half-density of g up to the constant $1/\sqrt{\pi}$ and transforms as a section of the half-density bundle under reparameterizations $\varphi \in \text{Diff}_+(M)$. \square

T2. Closed-Form Thomas–Wigner Rotation

Theorem 2 (T2). *Two sequential binary projective measurements at visibilities V_1, V_2 with unit axes $\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2$ separated by angle θ produce a composite boost-plus-rotation whose Thomas–Wigner angle Ω satisfies*

$$\tan(\Omega/2) = \frac{V_1 V_2 \gamma_1 \gamma_2 \sin \theta}{(\gamma_1 + 1)(\gamma_2 + 1) + V_1 V_2 \gamma_1 \gamma_2 \cos \theta},$$

equivalently in rapidity form: $\tan(\Omega/2) = \tanh(\eta_1/2) \tanh(\eta_2/2) \sin \theta / [1 + \tanh(\eta_1/2) \tanh(\eta_2/2) \cos \theta]$. The rotation axis is oriented along $\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2$.

Proof. By Burns–Greenfield–Dressel [2], the $\text{SL}(2, \mathbb{C})$ lift of a binary measurement at visibility V along axis $\hat{\mathbf{n}}$ is $L(\eta, \hat{\mathbf{n}}) = \cosh(\eta/2) I + \sinh(\eta/2)(\hat{\mathbf{n}} \cdot \boldsymbol{\sigma})$. Writing $c_i = \cosh(\eta_i/2)$, $s_i = \sinh(\eta_i/2)$, composition $L_2 L_1$ via $\sigma_i \sigma_j = \delta_{ij} I + i\varepsilon_{ijk} \sigma_k$ gives $L_2 L_1 = AI + \mathbf{B} \cdot \boldsymbol{\sigma} + i\mathbf{C} \cdot \boldsymbol{\sigma}$ with

$$A = c_1 c_2 + s_1 s_2 \cos \theta, \quad \mathbf{B} = c_2 s_1 \hat{\mathbf{n}}_1 + c_1 s_2 \hat{\mathbf{n}}_2, \quad \mathbf{C} = s_1 s_2 (\hat{\mathbf{n}}_2 \times \hat{\mathbf{n}}_1),$$

with $\mathbf{B} \cdot \mathbf{C} = 0$. The polar decomposition $L_2 L_1 = HU$ with $U = \cos(\Omega/2)I - i \sin(\Omega/2)(\hat{\mathbf{u}} \cdot \boldsymbol{\sigma})$ yields $\hat{\mathbf{u}} = -\mathbf{C}/|\mathbf{C}| = (\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2)/|\hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2|$ and $\tan(\Omega/2) = |\mathbf{C}|/A = s_1 s_2 \sin \theta / (c_1 c_2 + s_1 s_2 \cos \theta)$. Substituting $\tanh(\eta/2) = V\gamma/(\gamma + 1)$ gives both stated forms.

Numerical check at $V_1 = V_2 = 0.9$, $\theta = \pi/2$: $\gamma = 1/\sqrt{0.19} \approx 2.2942$, giving $\Omega \approx 42.90^\circ$, verified by direct SU(2) matrix polar decomposition. \square

T3. SLD Saturation for Binary Projective Qubit Measurements

Theorem 3 (T3). *For a qubit state $\rho(\theta)$ with Bloch vector $\mathbf{r}(\theta)$ of radius $r = |\mathbf{r}|$, a projective binary measurement along $\hat{\mathbf{n}}$ yields classical Fisher information $I_{\text{cl}}(\theta) = (\hat{\mathbf{n}} \cdot \partial_\theta \mathbf{r})^2 / (1 - V^2)$ where $V = \hat{\mathbf{n}} \cdot \mathbf{r}$. This saturates $F_Q(\theta) = |\partial_\theta \mathbf{r}|^2 + (\mathbf{r} \cdot \partial_\theta \mathbf{r})^2 / (1 - r^2)$ if and only if $\hat{\mathbf{n}}$ is parallel to the maximum-eigenvalue eigendirection of the symmetric logarithmic derivative [11].*

Proof. Decompose $\partial_\theta \mathbf{r} = a\hat{\mathbf{r}} + b\hat{\mathbf{m}}$ with $\hat{\mathbf{m}} \perp \hat{\mathbf{r}}$. For $\hat{\mathbf{n}} = \cos \beta \hat{\mathbf{r}} + \sin \beta \hat{\mathbf{m}}$,

$$I_{\text{cl}}(\beta) = \frac{(a \cos \beta + b \sin \beta)^2}{1 - r^2 \cos^2 \beta}.$$

Setting $dI_{\text{cl}}/d\beta = 0$ yields $\tan \beta_{\text{opt}} = b(1 - r^2)/a$; direct substitution confirms $I_{\text{cl}}(\beta_{\text{opt}}) = a^2/(1 - r^2) + b^2 = F_Q$. The SLD $L_\theta = \ell_0 I + \boldsymbol{\ell} \cdot \boldsymbol{\sigma}$ with $\boldsymbol{\ell} = (a/(1 - r^2))\hat{\mathbf{r}} + b\hat{\mathbf{m}}$ has eigendirection $\boldsymbol{\ell}/|\boldsymbol{\ell}|$ at angle β satisfying $\tan \beta = b(1 - r^2)/a = \tan \beta_{\text{opt}}$, identifying the saturating axis with the SLD eigendirection. \square

T4. Uniqueness of the Gudermannian Bridge

Theorem 4 (T4). *The Gudermannian function $\text{gd}(\eta) = \arcsin(\tanh \eta) = 2 \arctan(\tanh(\eta/2))$ is the unique smooth diffeomorphism $\psi: \mathbb{R} \rightarrow (-\pi/2, \pi/2)$ with $\psi(0) = 0$ satisfying $\tanh(\eta) = \sin(\psi(\eta))$ for all $\eta \in \mathbb{R}$.*

Proof. Differentiate the intertwining identity: $\text{sech}^2(\eta) = \cos(\psi(\eta)) \cdot \psi'(\eta)$. At $\psi = \text{gd}$, $\cos(\psi) = \text{sech}(\eta)$, so $\psi'(\eta) = \text{sech}(\eta)$. Integration with $\psi(0) = 0$ gives $\psi(\eta) = \int_0^\eta \text{sech}(u) du = \text{gd}(\eta)$. Uniqueness follows from ODE existence-uniqueness for first-order smooth initial-value problems. \square

T5. Binary Coarse-Graining Retention Closed Form

Theorem 5 (T5). *For a qubit state with Bloch-vector magnitude $r \in [0, 1]$ undergoing transverse rotation (constant r , unit angular speed of $\hat{\mathbf{r}}$, so $F_Q = r^2$), the ratio of isotropically averaged binary Fisher information to quantum Fisher information is*

$$R(r) = \frac{2r + (r^2 - 1) \ln \frac{1+r}{1-r}}{4r^3}.$$

$R(r)$ interpolates monotonically from $R(0) = 1/3$ to $R(1) = 1/2$, with series $R(r) = 1/3 + r^2/15 + O(r^4)$.

Proof. Isotropic averaging $(1/4\pi) \iint I_{\text{cl}}(\hat{\mathbf{n}}) \sin \alpha d\alpha d\beta$ reduces via β -integration ($\int_0^{2\pi} \cos^2 \beta d\beta = \pi$) to

$$\langle I_{\text{cl}} \rangle_{S^2} = \frac{r^2}{4} \int_0^\pi \frac{\sin^3 \alpha}{1 - r^2 \cos^2 \alpha} d\alpha = \frac{r^2}{4} \int_0^1 \frac{2(1 - u^2)}{1 - r^2 u^2} du,$$

using $u = \cos \alpha$ and exploiting the symmetry of the integrand. Partial fractions yield

$$\int_0^1 \frac{2(1-u^2)}{1-r^2u^2} du = \frac{2}{r^2} - \frac{1-r^2}{r^3} \ln \frac{1+r}{1-r}.$$

Dividing by 4 (absorbing the $r^2/4 \cdot 1/r^2 = 1/4$) gives $R(r)$. The series expansion and endpoint limits follow from $\ln((1+r)/(1-r)) = 2r + 2r^3/3 + 2r^5/5 + O(r^7)$ and $(1-r^2) \ln(1-r^2) \rightarrow 0$ as $r \rightarrow 1$.

Monotonicity follows from $R'(r) = [(3-r^2) \ln((1+r)/(1-r)) - 6r]/(4r^4)$, which is strictly positive on $(0, 1)$ because $(3-r^2) \ln((1+r)/(1-r)) - 6r = 8r^5/15 + O(r^7)$ by Taylor expansion, with every coefficient of $\ln((1+r)/(1-r))$ above leading order strictly exceeding the corresponding coefficient of $6r/(3-r^2)$. \square

Part B: Observer–Manifold Theorems

T6. Futility Theorem via Lawvere’s Fixed-Point Theorem

Theorem 6 (T6, abstract). *Let \mathcal{C} be a cartesian closed category, $A, B \in \text{Obj}(\mathcal{C})$, and $f: B \rightarrow B$ a morphism with no fixed point. Then no morphism $e: A \rightarrow B^A$ is point-surjective.*

Proof. Lawvere [8] in contrapositive form. Suppose e is point-surjective; define $g: A \rightarrow B$ by $g(a) = f(e(a)(a))$. Point-surjectivity yields a^* with $e(a^*) = g$, whence $g(a^*) = f(e(a^*)(a^*)) = f(g(a^*))$, a fixed point of f , contradicting the hypothesis. See Higuchi [7] for a survey. \square

Theorem 7 (T6, physical specialization). *Take $\mathcal{C} = \mathbf{Set}$. Let A be the observation-history space of an observer with bounded information-production rate; let $B = [1, \infty)$, the range of γ^2 field values on the binary Fisher–Rao manifold. Define $f(x) = x+1$ (fixed-point-free). Then no point-surjective $e: A \rightarrow B^A$ exists. In particular, the region where $\gamma^2(V) \rightarrow \infty$ is categorically inaccessible to any embedded observer.*

T7. Energy–Precision Uncertainty Relation

Theorem 8 (T7). *For N i.i.d. binary projective measurements at visibility V and temperature T , with outcomes recorded in finite-capacity classical memory and erased after readout, any unbiased estimator $\hat{\theta}$ satisfies*

$$E \cdot \text{Var}(\hat{\theta}) \geq kT \ln 2 \cdot (1 - V^2) = \frac{kT \ln 2}{\gamma^2(V)},$$

saturating the Landauer cost $kT \ln 2$ at $V = 0$.

Proof. Cramér–Rao: $\text{Var}(\hat{\theta}) \geq 1/(N\gamma^2(V))$ [11]. Landauer [12, 13]: $E \geq NkT \ln 2$. Multiplying the two bounds gives the stated inequality. \square

Corollary 9 (Planckian operating point). *At $V_P = \sqrt{1 - \ln 2} \approx 0.5539$, $\gamma^2(V_P) = 1/\ln 2 \approx 1.4427$ and $E \cdot \text{Var}(\hat{\theta}) \geq kT(\ln 2)^2 \approx 0.48kT$, which is a fraction $\ln 2 \approx 0.69$ of the Landauer cost.*

Part C: Symmetry and Classification

T8. Form-Invariance of the Conformal Equivalence under the Möbius Action

Theorem 10 (T8). *The conformal equivalence $ds_{\text{BK}}^2 = 4\gamma^2(V) ds_{\text{Bures}}^2$ on the qubit Bloch ball is form-invariant under the $\text{SL}(2, \mathbb{C})$ Möbius action $V \mapsto V' = (V + \xi)/(1 + V\xi)$: the equation holds*

in both the original and transformed coordinates with the same functional form $4\gamma^2$ evaluated at the new point.

Proof. The Bures and Beltrami–Klein metrics on the qubit Bloch ball are rank-2 symmetric tensor fields. Their conformal relationship is a tensor equation established in [1]. The $\text{SL}(2, \mathbb{C})$ Möbius action is a diffeomorphism of the Bloch ball [9, 10]. Under this action, $\gamma^2(V)$ transforms via $\gamma^2(V') = \gamma^2(V)\gamma^2(\xi)(1+V\xi)^2$ (direct computation from $1-V'^2 = (1-V^2)(1-\xi^2)/(1+V\xi)^2$), and the metric components transform by the standard tensor rule. The equation $ds_{\text{BK}}^2 = 4\gamma^2 ds_{\text{Bures}}^2$ therefore holds in the transformed coordinate with γ^2 evaluated at V' . This is general covariance (form-invariance), not pointwise invariance. \square

T9. Classification of Unsharp Binary Qubit Effects

Theorem 11 (T9). *Two-outcome binary qubit effects $E_{\pm} = (I \pm V\hat{\mathbf{n}} \cdot \boldsymbol{\sigma})/2$ with visibility $V \in [0, 1]$ and axis $\hat{\mathbf{n}} \in S^2$ are classified, up to $\text{SU}(2)$ equivalence by unitary conjugation, by the single parameter V . The sharp subfamily $V = 1$ consists of rank-1 projective POVMs; the unsharp subfamily $V < 1$ consists of effects with $\text{rank}(E_{\pm}) = 2$.*

Proof. $\text{SU}(2)$ acts on the Pauli vector by the spinor lift of $\text{SO}(3)$: $U(\hat{\mathbf{n}} \cdot \boldsymbol{\sigma})U^{-1} = (R\hat{\mathbf{n}}) \cdot \boldsymbol{\sigma}$. Conjugation sends $(V, \hat{\mathbf{n}})$ to $(V, R\hat{\mathbf{n}})$, preserving V . The $\text{SO}(3)$ -orbit of $\hat{\mathbf{n}}$ on S^2 is the whole sphere; the quotient $\mathcal{E}/\text{SU}(2)$ is parameterized by V . At $V = 1$, E_+ has eigenvalues $\{1, 0\}$ so $\text{rank}(E_+) = 1$; at $V < 1$, eigenvalues $(1 \pm V)/2 \in (0, 1)$, so $\text{rank}(E_+) = 2$. The Weyl obstruction at $N \geq 3$ [1] prevents extension of this classification to higher-dimensional systems via a single conformal factor. \square

T10. Modular Flow as Imaginary-Time $\text{SL}(2, \mathbb{C})$ Boost

Theorem 12 (T10). *For a qubit diagonal state $\rho = \text{diag}(p, 1-p)$ with $p \in (0, 1)$, the modular operator satisfies*

$$\rho^{it} = e^{i\phi(t)} \cdot \exp(it\eta\sigma_z)$$

where $\eta = \frac{1}{2} \log \frac{p}{1-p} = \text{arctanh}(2p-1)$ and $\phi(t) = (t/2) \log(p(1-p))$ is a state-dependent global phase, linear in t . Modulo this phase, the modular flow at modular time t corresponds to an $\text{SL}(2, \mathbb{C})$ boost along the z -axis with spinor rapidity $\zeta = 2it\eta$.

Proof. $\rho^{it} = \text{diag}(e^{it \log p}, e^{it \log(1-p)})$. Factor the mean phase $\phi(t) = (t/2)(\log p + \log(1-p)) = (t/2) \log(p(1-p))$ and the traceless part $\psi(t) = (t/2)(\log p - \log(1-p))$; then $\rho^{it} = e^{i\phi(t)} \cdot \exp(i\psi(t)\sigma_z)$. Identifying $\psi(t) = t\eta$ with $\eta = \text{arctanh}(2p-1)$ gives the stated form. The real-boost spinor $B(\zeta, \hat{\mathbf{z}}) = \exp((\zeta/2)\sigma_z)$ matches $\exp(it\eta\sigma_z)$ when $\zeta = 2it\eta$. \square

T11. SLOCC Covariance of Single-Qubit Binary Visibility

Theorem 13 (T11a). *Under a single-qubit SLOCC boost with parameter $\xi \in (-1, 1)$ along axis $\hat{\mathbf{n}}$, the visibility along that axis transforms by Einstein addition $V' = (V + \xi)/(1 + V\xi)$, and the squared Lorentz factor transforms as*

$$\gamma^2(V') = \gamma^2(V) \cdot \gamma^2(\xi) \cdot (1 + V\xi)^2.$$

Proof. A SLOCC boost is $A(\xi, \hat{\mathbf{n}}) = \cosh(\eta/2)I + \sinh(\eta/2)(\hat{\mathbf{n}} \cdot \boldsymbol{\sigma})$ with $\xi = \tanh \eta$. Its action on the Bloch component along $\hat{\mathbf{n}}$ is Einstein addition $V \mapsto (V + \xi)/(1 + V\xi)$. Direct algebra yields $1 - V'^2 = (1 - V^2)(1 - \xi^2)/(1 + V\xi)^2$, hence $\gamma^2(V') = (1 + V\xi)^2 / [(1 - V^2)(1 - \xi^2)] = \gamma^2(V)\gamma^2(\xi)(1 + V\xi)^2$. \square

Conjecture 14 (T11b, multi-qubit). *For n -qubit tensored SLOCC $A_1 \otimes \cdots \otimes A_n$, the binary coarse-graining visibility transforms covariantly with a weight factorizing into single-qubit factors.*

Part D: Quantum Gravity Connections

T12. Complexity–Fisher–Lorentz Correspondence

Theorem 15 (T12, scaling correspondence). *For a holographic CFT dual to a boosted black-brane background in $\text{AdS}_d/\text{CFT}_{d-1}$ with $d \in \{2, 3, 4\}$, the ratio of boosted-to-rest-frame subregion-CV complexity scales as the Fisher information of a binary measurement along the boost axis:*

$$\frac{C_V(\text{boost } \eta)}{C_V(0)} = \kappa_d \cdot \gamma^2(V) = \kappa_d \cdot I(V),$$

where $V = \tanh \eta$ and $\kappa_d > 0$ are dimension-specific universal constants.

Proof sketch. The $\text{SL}(2, \mathbb{C})$ boost acts on Hermitian 2×2 matrices $X \mapsto LXL^\dagger$ preserving $\det(X)$. The Hubeny–Rangamani–Takayanagi surface transforms covariantly. Daryaei Goki et al. [5] showed the enclosed-volume ratio scales as γ^2 universally across $d = 2, 3, 4$, establishing $C_V(\eta)/C_V(0) = \kappa_d \gamma^2(V)$ with κ_d a dimension-specific constant. The Fisher information along the boost axis is $\gamma^2(V)$ by the Fisher–Lorentz identity [1]. Both scale as γ^2 under the same $\text{SL}(2, \mathbb{C})$ action. \square

Conjecture 16 (T12b, complexity–Fisher identity). *There exists a choice of bulk subregion, boundary observable, and normalization convention in each dimension $d \in \{2, 3, 4\}$ such that $\kappa_d = 1$, making T12 an identity rather than a scaling equivalence.*

T13. γ^2 Page Curve for Black Hole Evaporation

Theorem 17 (T13). *The fine-grained entropy of Hawking radiation satisfies*

$$S_{\text{rad}}(t) = \frac{S_{\text{BH}}}{2} (1 - V^2(t)) = \frac{S_{\text{BH}}}{2\gamma^2(V(t))},$$

where $V(t)$ is the visibility of the inside-outside horizon binary measurement, monotonically decreasing from $V(0) = +1$ through $V(t_{\text{Page}}) = 0$ to $V(t_{\text{total}}) = -1$.

Proof sketch. Gómez [6] showed the quantum Fisher information of Hawking radiation scales as $1/S_{\text{BH}}$. The Hawking-pair inside-outside structure provides a natural qubit [16, 17]. The Page curve endpoints $S_{\text{rad}} = 0$ at $t = 0, t_{\text{total}}$ and peak $S_{\text{BH}}/2$ at t_{Page} (where $V = 0$) constrain the functional form to $S_{\text{rad}}(t) = (S_{\text{BH}}/2)(1 - V^2(t))$ as a natural smooth even function of V satisfying these boundary conditions. Uniqueness among such functions is not claimed; this is a plausibility argument, not a derivation. \square

Part E: Partial Result

T14. Dimensional Selection of 3+1D Spacetime

Theorem 18 (T14, partial result). *Under assumptions (A1) $N = 2$ qubit, (A2) conformal equivalence $ds_{\text{BK}}^2 = 4\gamma^2 ds_{\text{Bures}}^2$, (A3) $\text{SL}(2, \mathbb{C})$ qubit measurement symmetry, the emergent Lorentzian spacetime signature is $(1, 3)$ uniquely via the exceptional isomorphism catalog $\text{Spin}^+(p, q) \cong \text{SL}(2, \mathbb{K})$ at $(p, q) = (2, 1), (3, 1), (5, 1)$ combined with the Stueckelberg requirement $\mathbb{K} = \mathbb{C}$ [15].*

Remark 19 (Explicit gap). This argument derives the signature *given* a spacetime; it does not derive the existence of spacetime from the measurement structure. This is a load-bearing gap, acknowledged as such. The theorem is presented as a partial result pending resolution of the spacetime-existence question.

Scope and Boundaries

The fourteen theorems are proved or sketched within the single-qubit ($N = 2$) binary measurement framework. Three structural boundaries delimit their reach:

The Weyl obstruction. The conformal equivalence $ds_{\text{BK}}^2 = 4\gamma^2 ds_{\text{Bures}}^2$ holds exclusively for $N = 2$. Extension to $N \geq 3$ requires Weyl-obstruction-aware modifications that may preserve some but not all of the theorems above.

Spacetime existence. Theorem T14 derives the signature $(1, 3)$ conditional on the existence of a spacetime manifold. The derivation of spacetime existence itself from the qubit measurement structure remains an open foundational problem in quantum gravity. T12 and T13 provide indirect evidence for the measurement-to-spacetime link but do not constitute a derivation.

Multi-qubit SLOCC. Theorem T11a is proved for single qubits. The conjectured factorization for tensored SLOCC (Conjecture T11b) is stated but unproved.

Declarations

Competing interests. The author declares no competing interests. Provisional patent applications covering methods for qubit calibration drift detection, adaptive measurement-axis alignment, and Planckian-visibility sensor apparatus, derived from theorems in this framework, have been filed. Patents pending.

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Data availability. No datasets were generated or analyzed. All derivations are symbolic.

AI-tool disclosure. Computational and large-language-model tools were used for literature discovery and manuscript preparation. All mathematical claims, theorem statements, and proof constructions are the author's own work.

Priority statement. Theorems T1 through T14 as stated in this document are claimed as original work by Bharath G. Srivats as of the first Zenodo upload date of this concept DOI series. Theorems T1 through T11 are proved in full; T12 and T13 are proof sketches; T14 is a partial result with an explicit load-bearing gap, acknowledged in the text.

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